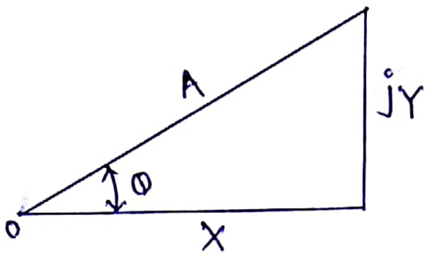
UNIT-05: COMPLEX NUMBER

POLAR FORM REPRESENTATION OF COMPLEX NO:-

$Z = A\angle\phi$



$A^2 = x^2 + y^2$

$= \sqrt{x^2+y^2}$

Also $x = A.\cos\phi$

$y = A.\sin\phi$

Again $\phi = \tan^{-1}\frac{y}{x}$

CONVERTING BETWEEN RECTANGULA FORM & POLAR FORM

$6\angle 30^\circ = x + jY$

$x = A.\cos\phi \quad y = A.\sin\phi$

Therefore, $6\angle 30^\circ = (6\cos\phi) + j(6\sin\phi)$

$= (6\cos 30^\circ) + j(6\sin 30^\circ)$

$= (6 \times 0.866) + j(6 \times 0.5)$

$= 5.2 + j3$

RECTANGULAR TO POLAR CONVERSION:-

$5.2 + j3 = A\angle\phi$

where $A = \sqrt{(5.2)^2 + 3^2} = 6$

$\phi = \tan^{-1}\left(\frac{3}{5.2}\right) = 30^\circ$

Hence, $(5.2 + j3) = 6\angle 30^\circ$

✡ Multiplication in polar form & Divison in polar form.

$$Z_1 \times Z_2 = A_1 \times A_2 \angle \theta_1 + \theta_2$$

Example

$$6\angle 30^\circ \times 8\angle -45^\circ = 6\times 8 \angle 30^\circ + (-45^\circ)$$
$$= 48 \angle -15^\circ$$

DIVISION IN POLAR FORM:-

$$\frac{Z_1}{Z_2} = \left(\frac{A_1}{A_2}\right) \angle \theta_1 - \theta_2$$

Example:

$$\frac{4\angle 90^\circ}{2\angle 45^\circ} = \frac{4}{2} \angle 90^\circ - 45^\circ = 2\angle 45^\circ$$

Q. 1. If $z_1 = 16\angle -30^\circ$ and $z_2 = 5\angle 30^\circ$ then find:
(1) $z_1 + z_2$ (2) $z_1 - z_2$ (3) $z_1 \times z_2$ (4) z_1 / z_2 (5) $z_1^* - z_2^*$

Solve: $z_1 = 16\angle -30$ where $A = 16$ $\theta = -30^\circ$

$$A\angle\theta = A\cos\theta + jA\sin\theta$$
$$= 16\cos(-30)^\circ + j16\sin(-30)^\circ$$
$$= 16 \times 0.866 + j16 \times (-0.5)$$
$$= 13.856 - j8$$

$Z_2 = 5\angle 30$ where $A = 5$ $\theta = 30^\circ$

$$A\angle\theta = A(\cos\theta + j\sin\theta)$$
$$= 5(\cos 30^\circ + j\sin 30)$$
$$= 4.33 + j2.5$$

① $z_1 + z_2 = (13.856 - j8) + (4.33 + j2.5)$
$$= 18.18 - j5.5$$
$$= 19\angle -16.82$$

②

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(2) $Z_1 - Z_2 = (13.856 - j8) - (4.33 + j2.5)$

$= 9.526 - j10.5$

$= 14.17 \angle -47.78$

(3) $Z_1 \times Z_2 = (16\angle -30) \times (5\angle 30)$

$= 16 \times 5 \angle (-30) + (30)$

$= 80\angle 0$

(4) $\dfrac{Z_1}{Z_2} = \dfrac{(16\angle -30)}{(5\angle 30)} = \dfrac{16}{5} \angle -30 - 30$

$= 3.2 \angle -60^\circ$

(5) $Z_1^* - Z_2^*$

$Z_1^* = 13.856 + j8$

$Z_2^* = 4.33 - j2.5$

$\left\{ \begin{array}{l} Z = a + ib \\ \therefore Z_1^* = a - ib \end{array} \right.$

$Z_1^* - Z_2^* = (13.856 + j8) - (4.33 - j2.5)$

$= 9.526 + j5.5$

$= 10.99 \angle 30^\circ$

Q.2: The following three phasor are given $A = 5 + j5$, $B = 50\angle 40^\circ$, $C = 4 + j0$ than find:

(a) $\dfrac{AB}{C}$ (b) $\dfrac{BC}{A}$

Solution:- $A = 5 + j5$

$A = \sqrt{(5)^2 + (5)^2} = 7.07$

$\theta = \tan^{-1}\left(\frac{y}{x}\right) = \tan^{-1}\left(\frac{5}{5}\right) = 45^\circ$

$A\angle\theta = 7.07 \angle 45^\circ$

$C = 4 + j0$ $\quad A = \sqrt{(4)^2 + (0)^2} = 4$

$\theta = \tan^{-1}\left(\frac{0}{4}\right) = 0$

$A\angle\theta = 4\angle 0^\circ$

(1) $\frac{AB}{C} = \frac{(7.07\angle 45^\circ)(50\angle 40^\circ)}{4\angle 0^\circ} = \frac{7.07 \times 50 \angle 45^\circ + 40^\circ}{4\angle 0^\circ}$

$= \frac{353.5\angle 85^\circ}{4\angle 0^\circ}$

$= 88.375\angle 85^\circ$ — Ans.

(2) $\frac{BC}{A} = \frac{(50\angle 40^\circ)(4\angle 0^\circ)}{7.07\angle 45^\circ} = \frac{50 \times 4 \angle 40^\circ + 0^\circ}{7.07\angle 45^\circ}$

$= \frac{200\angle 40^\circ}{7.07\angle 45^\circ}$

$= \frac{200}{7.07}\angle 40^\circ - 45^\circ$

$= 28.28\angle -5^\circ$

Q.3 The following three phasor are given $A = 20 + j20$, $B = 30\angle -120^\circ$, $C = 10 + j0$ than find (a) $\frac{AB}{C}$ (b) $\frac{BC}{A}$